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Technical Note

1969-62

Frequency Domain Analysis
of a Class
of Nonlinear Networks

J. Gorski-Popiel

26 November 1969

Prepared under Electronic Systems Division Contract AF 19(628)-5167 by

Lincoln Laboratory

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

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LINCOLN LABORATORY

FREQUENCY DOMAIN ANALYSIS
OF A CLASS OF NONLINEAR NETWORKS

J. GORSKI-POPIEL

Group 62

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ABSTRACT

A method for analyzing in the frequency domain the performance of linear networks containing nonlinear resistors. This method is applied to the evaluation of the frequency performance of a reactively terminated mixer.

Accepted for the Air Force
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Chief, Lincoln Laboratory Office

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FREQUENCY DOMAIN ANALYSIS OF A CLASS OF NONLINEAR NETWORKS

The evaluation of the frequency performance of networks containing nonlinear devices is normally very involved. This report describes a procedure by which the frequency domain performance of networks containing devices that may be represented as nonlinear resistors can be evaluated with any desired accuracy. Since only general statements can be made about the application of this method to the whole class of networks considered, the detailed analysis of a balanced mixer, together with its frequency selective terminations, is presented as an example.

The Problem

The general problem considered is a network containing nonlinear resistors controlled by one or more independent voltages and/or currents embedded in a linear frequency invariant network (Fig. 1). Also, ports 1 and 2 are assumed to be the input and output ports of a 2-port N (dashed lines, Fig. 1). Later, the problem of embedding N in a general linear network including reactive elements is considered.

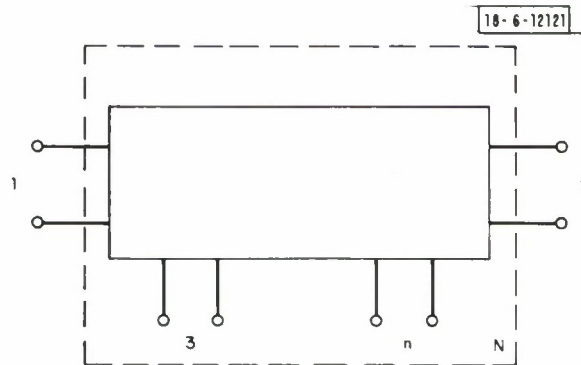


Fig. 1. Frequency invariant network containing nonlinear resistors.

Some Theoretical Considerations

The nonlinear resistors are assumed to be monotonic and representable, over the frequency band of interest, by some curve or family of curves (Fig. 2). The single curve type [i.e., Fig. 2(a)] will be a two-terminal device (e.g., a diode). If the device possesses three terminals, with one used in the fashion of a grid on a triode or gate on an FET, the multiple curve type description [Fig. 2(b)] is necessary. By analogy this scheme can be extended to cover more control parameters. The currents flowing through the nonlinear resistors will be referred to as $f_i(v)$, $f_i(v_1, v_2)$, etc., respectively (Fig. 2) to distinguish them from currents in the linear portion of the network.

Assume for the time being that only m two-terminal nonlinear resistors are present in the network. If these resistors are extracted to form m ports and if the n input ports are supplied with voltage sources V_{si} fed through resistors R_{si} , Fig. 3 results. It will be assumed for the time being that the V_{si} are DC voltages. The reason for this will become apparent later. NR_i stands for nonlinear resistance i . The current at the i^{th} output port is then $f_i(v_i)$ where v_i is the voltage across the port. Defining a $2n \times 2m$ transmission matrix between the n input and m nonlinear resistor ports with elements A, B, C, and D (each of which is an $n \times m$ matrix containing constants only),

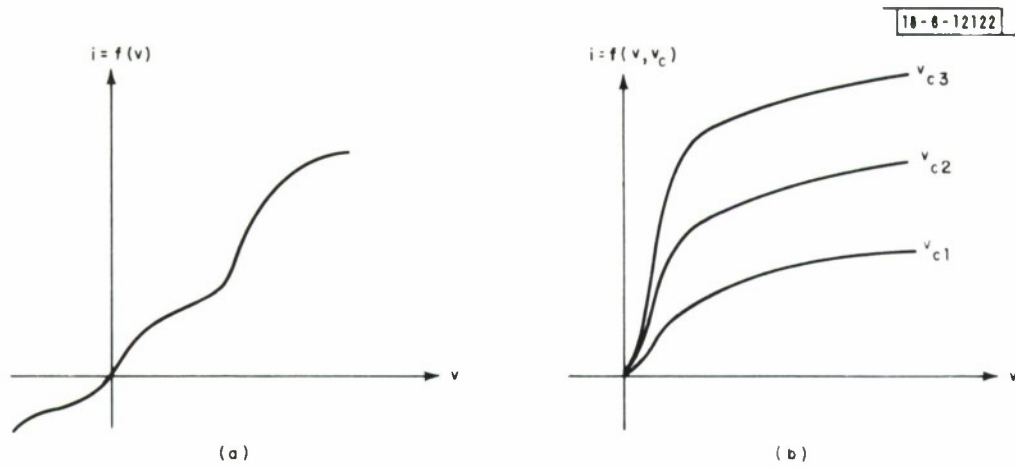


Fig. 2. Typical nonlinear resistance curves.

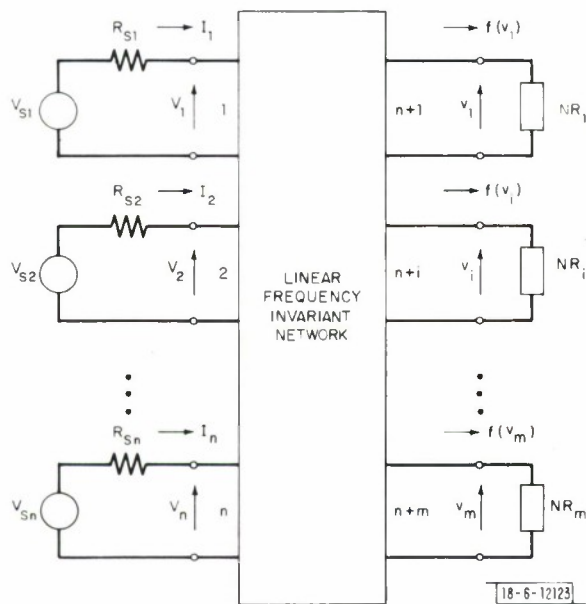


Fig. 3. $(n + m)$ -port linear network terminated in nonlinear resistors at the m -ports.

$$\underline{V}_S = (R_S C + A) \underline{x} + (R_S D + B) \underline{f}(\underline{v}) \quad (1)$$

where \underline{V}_S is an $n \times 1$ column vector, \underline{x} and $\underline{f}(\underline{v})$ are $m \times 1$ column vectors and R_S is an $n \times n$ diagonal matrix with resistor R_{Si} in the $i \times i$ position (for $i = 1, 2, \dots, n$).

If three-terminal nonlinear devices are now considered as well, then for each such device wherever there was one output port in Fig. 3 there will now be two. This scheme is illustrated in Fig. 4. It is assumed that no current flows into the control port. This can obviously be done with no loss in generality by merely assuming the resistance (if any) across this port as being

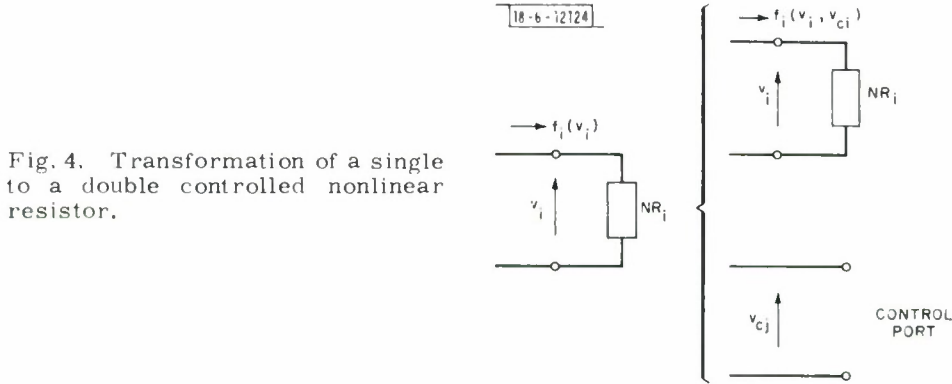


Fig. 4. Transformation of a single to a double controlled nonlinear resistor.

absorbed into the main network. So if there are k of these three-terminal devices present, the number of output ports will be increased to $(m + k)$. An equation essentially in the same form as that given in Eq. (1) will still hold; however, the transmission matrix elements will be $n \times (m + k)$ matrices and $\underline{f}(\underline{v})$ will be a function of two voltage variables, v_i and v_{ci} , for each of the three-terminal devices. If devices with more than two control parameters are used, the foregoing principles still hold merely increasing the order of the transmission matrices and appropriately changing $\underline{f}(\underline{v})$. Also, the controlling parameters need not be voltages, they may be any mixture of currents and voltages. No matter what the complexity of the devices, however, an equation of the form shown in Eq. (1) will always be obtainable by standard linear analysis techniques.

V - I curves for each of the nonlinear devices are assumed to be available, so for any one vector \underline{x} the vector $\underline{f}(\underline{v})$ may be evaluated. This may require some interpolation if three or more terminal nonlinear devices are present. From a knowledge of \underline{x} and $\underline{f}(\underline{v})$, by use of Eq. (1) the corresponding vector \underline{V}_S can be found. Also for any one vector \underline{x} a unique vector \underline{V}_S exists. The reverse is also true. Thus given sufficient space a large enough table can be constructed that will permit reverse interpolations, i.e., given a vector \underline{V}_S the corresponding vector \underline{x} can be found to within some specified accuracy.

Now consider Fig. 2(a). For any value of v , v_i a corresponding unique value $f(v_i)$ exists. Since $f(v_i)$ is a current, a resistance $R_i = v_i/f(v_i)$ may be defined. Again for a given v_i , R_i is unique. So from a given V - I curve, R_i can be easily found for each v . The same is quite obviously true of a family of curves [Fig. 2(b)]; however, here two variables v_i and v_{ci} are required to define R_i .

So for any given vector, \underline{V}_S , each nonlinear element can be replaced by its corresponding value, R_i . In effect, for each value of \underline{V}_S the entire network can be replaced by a purely positive linear network. If we consider the 2-port N (Fig. 1) and assume that all its V_{Si} are on ports other than 1 and 2, then for each value of \underline{V}_S one can write down any of the 2-port matrices by

straightforward linear network analysis methods. Consider the h-matrix; then

$$\begin{aligned} v_1 &= h_{11}(\underline{V}_s) i_1 + h_{12}(\underline{V}_s) v_2 \\ i_2 &= h_{21}(\underline{V}_s) i_1 + h_{22}(\underline{V}_s) v_2 \end{aligned} \quad (2)$$

as \underline{V}_s is changed to some new value, the h-parameters will also change. This dependence is implied by the notation used in Eq. (2).

So far it has been assumed that the elements of \underline{V}_s are DC voltages. But the main aim of this paper is to investigate the frequency performance of the networks considered. The foregoing discussion can be applied to this aim in the following manner. Assume that each of the voltages in \underline{V}_s is some sinusoidally varying signal. The frequency of each of these voltages used need not be the same. It will, however, be assumed that the magnitude of one V_{si} is far larger than that of all the others. The reason for this assumption will become apparent later. Let this dominant voltage be denoted by V_{so} and its frequency by f_o . Now let V_{so} and all the other V_{si} be sampled over one complete cycle of V_{so} . Each set of samples will determine one complete vector \underline{V}_s and hence one value for each of the h-parameters. If n samples are taken, this will determine the n different values each of the h-parameters assumes over one cycle of V_{so} . Since V_{so} was assumed to be dominant, this pattern will be repeated to a high degree of accuracy over each cycle of V_{so} . This scheme is illustrated in Fig. 5, taking $h_{12}(\underline{V}_s)$ as an example. Of course, all the other V_{si} present are also sampled at the same rate as V_{so} . The envelope of the resultant magnitudes of $h_{ij}(\underline{V}_s)$, ($i, j = 1, 2$) represents the variation of these parameters in time over

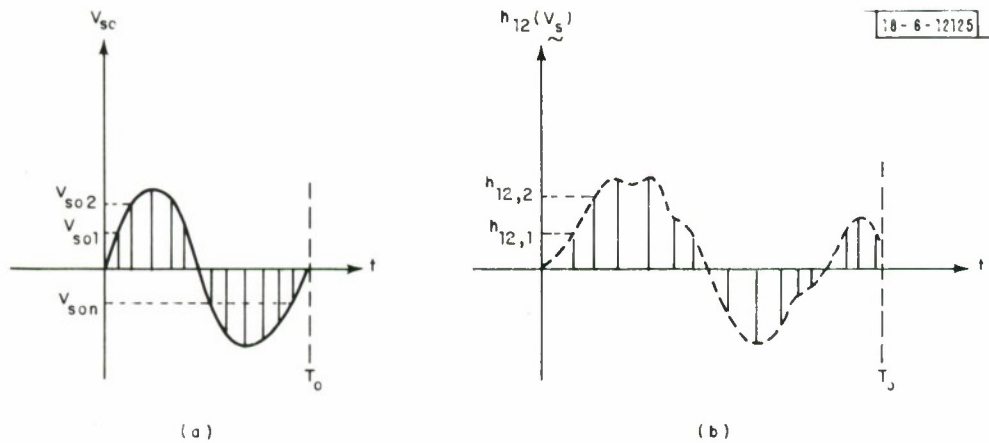


Fig. 5. (a) V_{so} input waveform, (b) possible resultant $h_{12}(\underline{V}_s)$ waveform.

a period T_o . The reason for choosing one dominant V_{si} is now apparent. If more than one V_{si} were dominant the periodicity of the h-parameters could well become indeterminate, at least in a general case. In many practical networks the assumption that one drive is dominant as far as the nonlinearities are concerned is founded. The example to be considered will illustrate this. At this stage, cases have to be considered on an individual basis, especially if the dominance of only one V_{si} can no longer be assumed. In this case, each of the four h_{ij} parameters can be expanded in a Fourier series with a fundamental period ω_o . Thus

$$h_{11}(t) = \sum_{i=0}^{\infty} H_{11,i} \exp[ji\omega_0 t] \quad (3a)$$

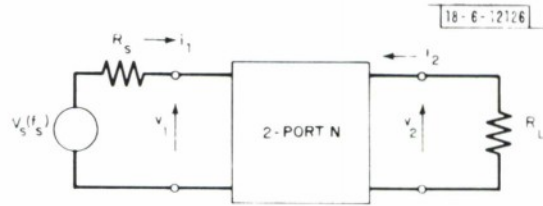
$$h_{12}(t) = \sum_{i=0}^{\infty} H_{12,i} \exp[ji\omega_0 t] \quad (3b)$$

$$h_{21}(t) = \sum_{i=0}^{\infty} H_{21,i} \exp[ji\omega_0 t] \quad (3c)$$

$$h_{22}(t) = \sum_{i=0}^{\infty} H_{22,i} \exp[ji\omega_0 t] \quad (3d)$$

So the problem posed at the beginning has now been reduced to a 2-port problem with a set of h-parameters given in Eq. (3). The amplitudes and fundamental frequency of these parameters is determined by V_{s0} . A number of other V_{si} may also be present. It has to be noted that any DC voltage supplies can also be represented by a V_{si} . These DC voltages may be any required magnitude, comparable or even greater than V_{s0} , since they will not introduce a periodic variation.

Fig. 6. The 2-port N shown in Fig. 4 with source and load terminations.



In the most general case, the 2-port considered will be supplied by some voltage V_s at frequency f_s (Fig. 6) on port 1 and work into some load R_L . The voltages and currents on the two sides of the network will have frequency components at $(f_s \pm kf_0)$ for $-\infty < k < \infty$, so denoting $|v_{2\ell}| \exp[j \arg(v_{2\ell})]$ by $V_{2\ell}$ and $|i_{1m}| \exp[j \arg(i_{1m})]$ by I_{1m} ,

$$v_2(t) = \sum_{\ell=-\infty}^{\infty} V_{2\ell} \exp[j(\omega_s + \ell\omega_0) t] \quad (4a)$$

$$i_1(t) = \sum_{m=-\infty}^{\infty} I_{1m} \exp[j(\omega_s + m\omega_0) t] \quad (4b)$$

Similar expressions hold for the other parameters. Thus from Eqs. (2), (3) and (4):

$$\begin{aligned}
v_1(t) &= \sum_{i=0}^{\infty} H_{11,i} \exp[ji\omega_0 t] \times \sum_{m=-\infty}^{\infty} I_{1m} \exp[j(\omega_s + m\omega_0) t] \\
&+ \sum_{i=0}^{\infty} H_{12,i} \exp[ji\omega_0 t] \times \sum_{\ell=-\infty}^{\infty} V_{2\ell} \exp[j(\omega_s + \ell\omega_0) t] \\
&= \sum_{m=-\infty}^{\infty} \left(\sum_{i=0}^{\infty} H_{11,i} I_{1m} \exp[j(\omega_s + (m+i)\omega_0) t] + \exp[j(\omega_s + (m-i)\omega_0) t] \right) \\
&+ \sum_{\ell=-\infty}^{\infty} \left(\sum_{i=0}^{\infty} H_{12,i} V_{2\ell} \exp[j(\omega_s + (\ell+i)\omega_0) t] + \exp[j(\omega_s + (\ell-i)\omega_0) t] \right) \quad (5)
\end{aligned}$$

but since $v_1(t)$ will also be of the same form as the expressions in Eq. (4), i.e.,

$$v_1(t) = \sum_{n=-\infty}^{\infty} V_{1n} \exp[j(\omega_s + n\omega_0) t] \quad (6)$$

it follows from Eqs. (5) and (6) that

$$V_{1n} = \sum_{i=0}^{\infty} [H_{11,i}(I_{1,n-i} + I_{1,n+i}) + H_{12,i}(V_{2,n-i} + V_{2,n+i})] \quad (7)$$

and by the same argument

$$I_{2k} = \sum_{i=0}^{\infty} [H_{21,i}(I_{1,n-i} + I_{1,n+i}) + H_{22,i}(V_{2,n-i} + V_{2,n+i})] \quad (8)$$

Since $-\infty \leq n \leq \infty$ and $-\infty \leq k \leq \infty$, Eqs. (7) and (8) define a $2n \times 2k$ matrix with both dimensions stretching from $-\infty$ to $+\infty$. In every practical case the coefficients $H_{ab,i}$ will decrease as i increases, also [Eq. (4)] the coefficient V_{2k} and I_{1m} will decrease with an increase in k , so only a finite portion of the matrix has to be considered for any required accuracy. For example, assume that only the first three H-parameter coefficients need be considered, then Eq. (7) reduces to

$$\begin{aligned}
V_{1,n} &= H_{11,0} I_{1,n} + H_{11,1}(I_{1,n-1} + I_{1,n+1}) + H_{11,2}(I_{1,n-2} + I_{1,n+2}) \\
&+ H_{11,3}(I_{1,n-3} + I_{1,n+3}) + H_{12,0} V_{2,n} + H_{12,1}(V_{2,n-1} + V_{2,n+1}) \\
&+ H_{12,2}(V_{2,n-2} + V_{2,n+2}) + H_{12,3}(V_{2,n-3} + V_{2,n+3}) \quad (9)
\end{aligned}$$

Equation (8) will reduce to an analogous expression. If it is further assumed that only $V_{1,-2}, V_{1,-1}, \dots, V_{1,2}$ are of interest, then Eq. (9) may be rewritten in matrix form as

$$\begin{bmatrix} V_{1,-2} \\ V_{1,-1} \\ V_{1,0} \\ V_{1,1} \\ V_{1,2} \end{bmatrix} = \begin{bmatrix} H_{11,3} & H_{11,2} & H_{11,1} & H_{11,0} & H_{11,1} & H_{11,2} & H_{11,3} & 0 & 0 & 0 & 0 \\ 0 & H_{11,3} & H_{11,2} & H_{11,1} & H_{11,0} & H_{11,1} & H_{11,2} & H_{11,3} & 0 & 0 & 0 \\ 0 & 0 & H_{11,3} & H_{11,2} & H_{11,1} & H_{11,0} & H_{11,1} & H_{11,2} & H_{11,3} & 0 & 0 \\ 0 & 0 & 0 & H_{11,3} & H_{11,2} & H_{11,1} & H_{11,0} & H_{11,1} & H_{11,2} & H_{11,3} & 0 \\ 0 & 0 & 0 & 0 & H_{11,3} & H_{11,2} & H_{11,1} & H_{11,0} & H_{11,1} & H_{11,2} & H_{11,3} \end{bmatrix} \begin{bmatrix} I_{1,-5} \\ I_{1,-4} \\ I_{1,-3} \\ I_{1,-2} \\ I_{1,-1} \\ I_{1,0} \\ I_{1,1} \\ I_{1,2} \\ I_{1,3} \\ I_{1,4} \\ I_{1,5} \end{bmatrix} \quad (10)$$

$$\begin{bmatrix} H_{12,3} & H_{12,2} & H_{12,1} & H_{12,0} & H_{12,1} & H_{12,2} & H_{12,3} & 0 & 0 & 0 & 0 \\ 0 & H_{12,3} & H_{12,2} & H_{12,1} & H_{12,0} & H_{12,1} & H_{12,2} & H_{12,3} & 0 & 0 & 0 \\ 0 & 0 & H_{12,3} & H_{12,2} & H_{12,1} & H_{12,0} & H_{12,1} & H_{12,2} & H_{12,3} & 0 & 0 \\ 0 & 0 & 0 & H_{12,3} & H_{12,2} & H_{12,1} & H_{12,0} & H_{12,1} & H_{12,2} & H_{12,3} & 0 \\ 0 & 0 & 0 & 0 & H_{12,3} & H_{12,2} & H_{12,1} & H_{12,0} & H_{12,1} & H_{12,2} & H_{12,3} \end{bmatrix} \begin{bmatrix} V_{2,-5} \\ V_{2,-4} \\ V_{2,-3} \\ V_{2,-2} \\ V_{2,-1} \\ V_{2,0} \\ V_{2,1} \\ V_{2,2} \\ V_{2,3} \\ V_{2,4} \\ V_{2,5} \end{bmatrix}$$

Since the number of voltages and currents must be the same, only the portions of the matrices and column vectors inside the dashed lines need be considered. Equation (8) may, of course, also be written in a matrix form analogous to Eq.(10). Denoting the left-hand side of Eq. (10) by $\tilde{V}_{1,n}$, the two truncated column vectors on the right side by $\tilde{I}_{1,n}$ and $\tilde{V}_{2,n}$, and using a similar notation for the other relevant entities, Eqs. (7) and (8) may be rewritten as

$$\begin{bmatrix} \tilde{V}_{1,n} \\ \tilde{I}_{2,n} \end{bmatrix} = \begin{bmatrix} \tilde{H}_{11} & \tilde{H}_{12} \\ \tilde{H}_{21} & \tilde{H}_{22} \end{bmatrix} \begin{bmatrix} \tilde{I}_{1,n} \\ \tilde{V}_{2,n} \end{bmatrix} \quad (11)$$

Each of the \tilde{H} -parameters is an $n \times n$ matrix of constants, thus Eq. (11) describes the h-matrix of a linear 2 port with n input and n output ports, however, since each $\tilde{V}_{1,n}$, etc., is associated with one specific frequency, this 2 port differs fundamentally from a conventional linear network in that each port is associated with a distinct frequency different from all the others. Figure 7 is the pictorial representation of Eq. (11). Despite the fact that each port is associated with a different frequency, the network itself is linear and hence any transfer parameter can be

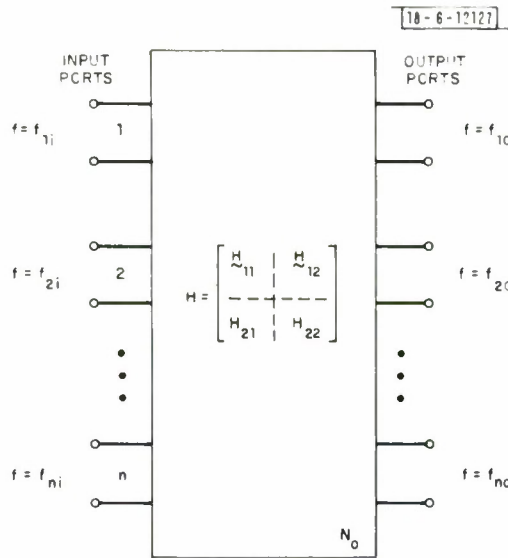


Fig. 7. Resultant linearization of the nonlinear 2-port N .

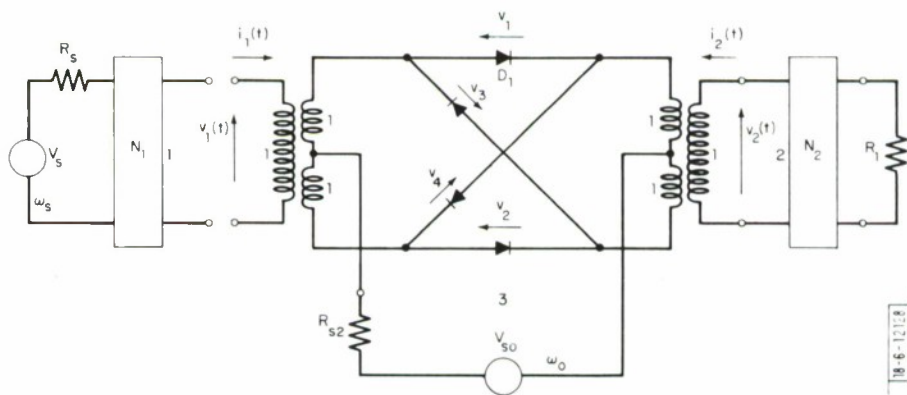


Fig. 8. Balanced mixer terminated in matching networks N_1 and N_2 .

computed using simply linear methods. Thus, for example, if the power transfer between input port 1 and output port i is required, this will be given by the $|S_{1,i}|^2$ where $S_{1,i}$ is the $1, i$ entry in the scattering matrix of network N_o . As far as the original 2-port N is concerned, $|S_{1,i}|^2$ defines the power transfer from port 1 at frequency $f_{1,i}$ to port 2 at frequency $f_{2,i}$. Any other parameters may be evaluated in a similar manner.

To recap, given a 2-port N containing nonlinear resistors, find the time performance of its h-parameters (or any other set for that matter), expand these in a Fourier series, collect the dominant coefficients to form Eq. (11) and the nonlinear 2-port problem is reduced to an equivalent linear $2n$ -port problem. For example;

The Balanced Ring Mixer

Problem: Investigate the frequency performance of the balanced mixer (Fig. 8) and determine the characteristics of matching networks placed on the input and output ports (networks N_1 and N_2), which will give the smallest insertion loss between the signals at source frequency and the required output frequency.

Let the RF signal at angular frequency ω_s be applied to port 1, the LO signal to port 3, and let port 2 be the output port at which the IF signal at angular frequency $(\omega_s - \omega_o)$ is extracted. The 3-coil transformers are assumed to be frequency invariant over the band of interest. Assuming the LO drive is dominant (this is a very fair assumption in the case) it will be found on inspection that

$$\begin{aligned} v_1 &= v_2 = V_{so} \\ v_3 &= v_4 = -V_{so} \end{aligned} \quad (12)$$

This simplifies matters very much since none of the interpolation procedures described at the beginning need be used. It is also assumed that the diodes are matched, and hence, have essentially the same V - i characteristics. These will be taken to be of the form shown in Fig. 9(a). With this assumption and the V_{so} drive as shown in Fig. 9(b), the resultant diode resistance performance against time will very nearly be that shown in Fig. 9(c). An additional degree of sophistication is added in Fig. 9(c) by assuming the rise and fall times are unequal. From Eq. (12) it follows that diodes D_1 and D_2 will have a resistance waveform of the type shown in Fig. 9(c). Let this resistance be denoted by $R(t)$. The resistances of diodes D_3 and D_4 will have the same shape as $R(t)$, however, they will be shifted by 180° with respect to $R(t)$. For this reason these impedances will be referred to as $R_-(t)$. By a straightforward Fourier analysis of Fig. 9(c), it follows that

$$R_+(t) = \frac{1}{2}(R_b + R_f) + \frac{1}{2}(R_b - R_f) \epsilon(t) \quad (13a)$$

$$R_-(t) = \frac{1}{2}(R_b + R_f) - \frac{1}{2}(R_b - R_f) \epsilon(t) \quad (13b)$$

where

$$\epsilon(t) = \sum_{n=1}^{\infty} \frac{1}{2} \left[\frac{\sin n\pi(\frac{\delta a}{T})}{n\pi(\frac{\delta a}{T})} + \frac{\sin n\pi(\frac{\delta b}{T})}{n\pi(\frac{\delta b}{T})} \right] \times \frac{\sin \frac{n\pi}{2}}{\frac{n\pi}{2}} \times \cos n\omega_o t \quad (14)$$

$$= \sum_{n=1}^{\infty} k(n) \frac{\sin \frac{n\pi}{2}}{n\pi/2} \cos n\omega_o t \quad (14a)$$

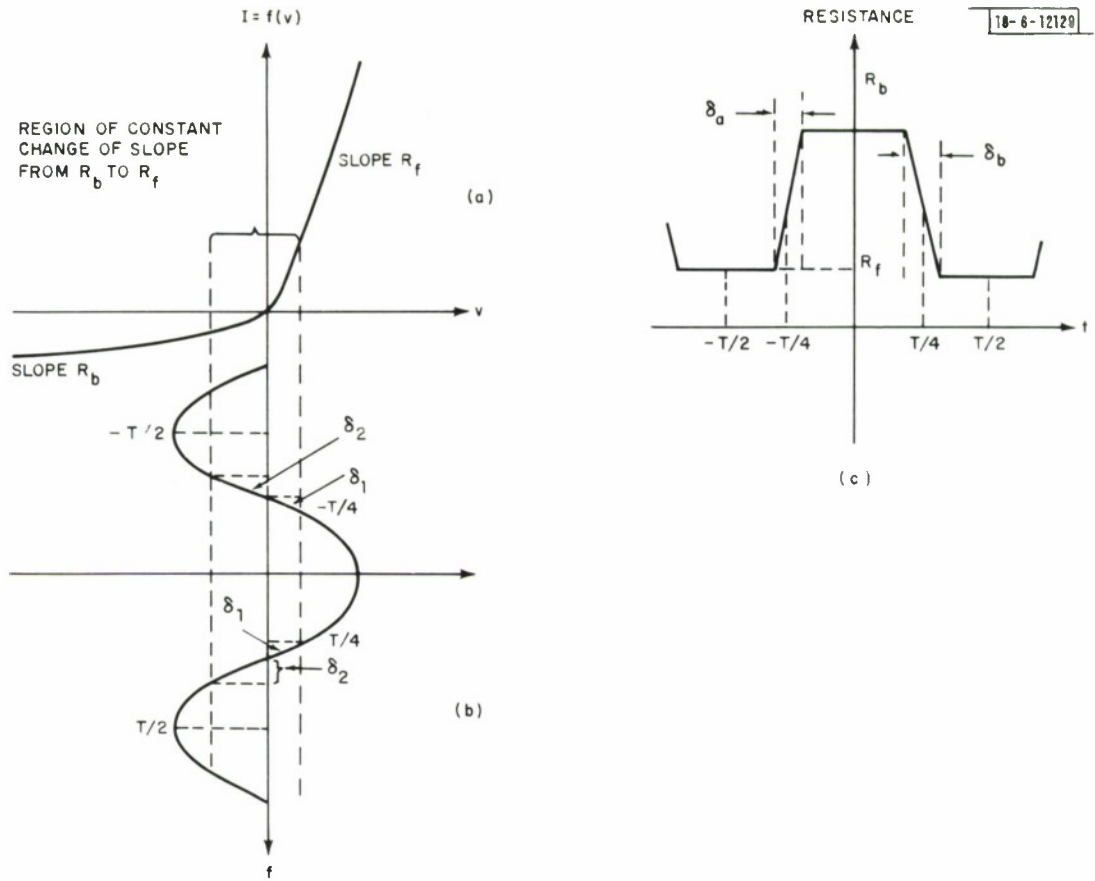


Fig. 9. (a) Diode characteristic, (b) V_{SO} cosine wave versus time, (c) resultant: resistance/time performance of a diode driven by the V_{SO} shown in (b).

This is a trapezoidal wave of ± 1 magnitude. It should be noted that if $\delta a = \delta b = 0$, $k(n) = 1$, then for all n and $\epsilon(t)$ reduces to

$$u(t) = \sum_{n=1}^{\infty} \frac{\sin \frac{n\pi}{2}}{\frac{n\pi}{2}} \cos n\omega_0 t \quad (15)$$

which is the Fourier expansion of a square wave varying between $+1$ and -1 .

An equivalent derivation on a conductance basis, assuming the two diode slopes to be G_b and G_f gives

$$G_+(t) = \frac{1}{2}(G_f + G_b) - \frac{1}{2}(G_f - G_b) \epsilon(t) \quad (16a)$$

$$G_-(t) = \frac{1}{2}(G_f + G_b) + \frac{1}{2}(G_f - G_b) \epsilon(t) \quad (16b)$$

$G_+(t)$ and $G_-(t)$ are defined in the same fashion as $R_+(t)$ and $R_-(t)$.

Assuming the instantaneous resistances (conductances) of the four diodes in Fig. 8 to be, respectively, R_1, R_2, R_3, R_4 (G_1, G_2, G_3, G_4) then the instantaneous h-matrix between ports 1 and 2 is given by

$$(h) = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} \left(\frac{1}{G_1 + G_3} + \frac{1}{G_2 + G_4} \right) & \frac{R_1 R_2 - R_3 R_4}{(R_1 + R_3)(R_2 + R_4)} \\ -\frac{R_1 R_2 - R_3 R_4}{(R_1 + R_3)(R_2 + R_4)} & \left(\frac{1}{R_1 + R_3} + \frac{1}{R_2 + R_4} \right) \end{bmatrix} \quad (17)$$

but $R_1 = R_2 = R_+(t)$ and $R_3 = R_4 = R_-(t)$, or vice versa.

A similar set of relations holds for the Y parameters. Using Eqs. (13), (16) and (17):

$$h_{11} = \frac{2}{G_b + G_f} \quad (18a)$$

$$h_{12} = -h_{21} = \left(\frac{R_b - R_f}{R_b + R_f} \right) \epsilon(t) \quad (18b)$$

$$h_{22} = \frac{2}{R_b + R_f} \quad (18c)$$

It is interesting to note, that h_{11} and h_{22} reduce to time invariant immittances. Using $\epsilon(t)$ in the form given in Eq. (14a)

$$h_{12} = -h_{21} = \frac{2}{\pi} \left(\frac{R_b - R_f}{R_b + R_f} \right) \left[k(1) \cos \omega_o t - \frac{1}{3} k(3) \cos 3\omega_o t + \dots \right. \\ \left. + \frac{(-1)^n}{(2n+1)} k(2n+1) \cos (2n+1) \omega_o t \dots \right]$$

it follows that

$$H_{11,o} = \frac{2}{G_b + G_f} \quad ; \quad H_{11,i} \Big|_{i \neq 0} = 0 \quad (19a)$$

$$H_{22,o} = \frac{2}{R_b + R_f} \quad ; \quad H_{22,i} \Big|_{i \neq 0} = 0 \quad (19b)$$

$$H_{12,o} = H_{21,o} = 0 \quad (19c)$$

$$H_{12,i} \Big|_{i \neq 0} = H_{21,i} \Big|_{i \neq 0} = \frac{2}{\pi} \left(\frac{R_b - R_f}{R_b + R_f} \right) \sum_{i=1}^{\infty} \frac{\sin \frac{i\pi}{2}}{i} k(i) \quad (19d)$$

Equation (19) completely defines all parameters in the 2n-port H-matrix. Table I gives a number of ratios for $k(n)$. In each instance the rise and fall times have been assumed equal (i.e., $\delta_a = \delta_b$). It should be noted that the fundamental component (i.e., $n = 1$) in each of the cases considered is very close to unity, and as n increases $k(n)$ departs from unity. So the higher order harmonics are attenuated to a larger degree than the fundamental. This fact will actually aid the mixer design. More will be said about this when the numerical results of this example are discussed.

TABLE I RATIOS FOR k(n)			
n	$\delta_a = \delta_b = 0.01$	$\delta_a = \delta_b = 0.05$	$\delta_a = \delta_b = 0.1$
1	0.999812	0.995865	0.983641
3	0.998433	0.963397	0.858397
5	0.995865	0.900316	0.636620
7	0.991945	0.810335	0.367884
9	0.986725	0.698648	0.109294

Assuming $\delta_a = \delta_b = 0$, the H-matrix of the same order as that in Eq. (10) becomes

$$\begin{bmatrix} V_{1,-2} \\ V_{1,-1} \\ V_{1,0} \\ V_{1,1} \\ V_{1,2} \\ \hline V_{2,-2} \\ V_{2,-1} \\ V_{2,0} \\ V_{2,1} \\ V_{2,2} \end{bmatrix} = \begin{bmatrix} H_{11,0,-2} & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -\frac{1}{3} & 0 \\ 0 & H_{11,0,-1} & 0 & 0 & 0 & 1 & 0 & 1 & 0 & -\frac{1}{3} \\ 0 & 0 & H_{11,0,0} & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & H_{11,0,1} & 0 & -\frac{1}{3} & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & H_{11,0,2} & 0 & -\frac{1}{3} & 0 & 1 & 0 \\ \hline 0 & 1 & 0 & -\frac{1}{3} & 0 & H_{22,0,-2} & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & -\frac{1}{3} & 0 & H_{22,0,-1} & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & H_{22,0,0} & 0 & 0 \\ \frac{1}{3} & 0 & 1 & 0 & 1 & 0 & 0 & 0 & H_{22,0,1} & 0 \\ 0 & -\frac{1}{3} & 0 & 1 & 0 & 0 & 0 & 0 & 0 & H_{22,0,2} \end{bmatrix} \begin{bmatrix} I_{1,-2} \\ I_{1,-1} \\ I_{1,0} \\ I_{1,1} \\ I_{1,2} \\ \hline I_{2,-2} \\ I_{2,-1} \\ I_{2,0} \\ I_{2,1} \\ I_{2,2} \end{bmatrix} \quad (20)$$

where $H_{11,0,k} = 1/(R_b + R_f)$ evaluated at $\omega = \omega_s + k\omega_o$ and $H_{22,0,k} = 1/(R_b + R_f)$ evaluated at $\omega = \omega_s + k\omega_o$. Equation (20) represents a 10-port passive reciprocal network with 5 input and 5 output ports. A number of important conclusions about the character of this 10-port can be drawn from Eq. (20). This can best be done by writing out some of the equations, thus:

$$V_{1,-2} = H_{11,0,-2} I_{1,-2} + V_{2,-1} - \frac{1}{3} V_{2,1} \quad (21a)$$

$$V_{1,-1} = H_{11,0,-1} I_{1,-1} + V_{2,-2} + V_{2,0} - \frac{1}{3} V_{2,2} \quad (21b)$$

$$V_{1,0} = H_{11,0,0} I_{1,0} + V_{2,-1} + V_{2,1} \quad (21c)$$

$$V_{1,1} = H_{11,0,1} I_{1,1} - \frac{1}{3} V_{2,-2} + V_{2,0} + V_{2,2} \quad (21d)$$

$$V_{1,2} = H_{11,0,2} I_{1,2} - \frac{1}{3} V_{2,-1} + V_{2,1} \quad (21e)$$

The frequency associated with any one port is determined by the voltage (or current suffix), thus $V_{1,i}$ is at a frequency $\omega_s + i\omega_o$, etc. So from Eq. (21) it follows that components at even multiples of the angular frequency ω_o on the output ports will produce components at odd multiples of ω_o at the input ports and vice versa. From Fig. 8 it can be seen on inspection that the input voltage components will contain the voltage $V_{1,0}$ (i.e., at ω_s), so there will be a current component $I_{1,0}$. This will result in only odd suffix voltages and currents on the output ports, which in turn will produce only even suffix voltages and currents on the input ports. As a result, only even harmonics of ω_o will be present on the input ports and only odd harmonics on the output ports. This has an important consequence because it implies that all $V_{1,i}$, $I_{1,i}$ for i odd, and $V_{2,i}$, $I_{2,i}$ for i even, will have zero amplitude. So two of the input ports and three of the output ports of the 10 ports are, for practical purposes, nonexistent. So they may be ignored. Since the mixer will be used (in this example) as a down converter, the component $V_{2,-1}$ is the required output. So from equations of the type given in Eq. (21) the input and output voltages, in decreasing order of magnitude, can be written as follows:

<u>Input Magnitudes</u>	<u>Output Magnitudes</u>
$V_{1,0}$	$V_{2,-1}$
$V_{1,-2}$	$V_{2,1}$
$V_{1,2}$	$V_{2,-3}$
$V_{1,-4}$	$V_{2,3}$
$V_{1,4}$	$V_{2,-5}$
.	.
.	.
.	.

and similarly for currents. Using these magnitudes as the new column vectors, a new 10-port matrix may be constructed by inspection of Eq. (10), thus

$$\begin{bmatrix} V_{1,0} \\ V_{1,-2} \\ V_{1,2} \\ V_{1,-4} \\ V_{1,4} \\ I_{2,-1} \\ I_{2,1} \\ I_{2,-3} \\ I_{2,3} \\ I_{2,-5} \end{bmatrix} \begin{bmatrix} H_{11,0,0} & 0 & 0 & 0 & 0 & 1 & 1 & -\frac{1}{3} & -\frac{1}{3} & \frac{1}{5} \\ 0 & H_{11,0,-2} & 0 & 0 & 0 & 1 & \frac{1}{3} & 1 & \frac{1}{5} & -\frac{1}{3} \\ 0 & 0 & H_{11,0,2} & 0 & 0 & -\frac{1}{3} & 1 & \frac{1}{5} & 1 & -\frac{1}{7} \\ 0 & 0 & 0 & H_{11,0,-4} & 0 & \frac{1}{3} & \frac{1}{5} & 1 & -\frac{1}{7} & 1 \\ 0 & 0 & 0 & 0 & H_{11,0,4} & \frac{1}{5} & -\frac{1}{3} & -\frac{1}{7} & 1 & \frac{1}{9} \\ \hline 1 & 1 & -\frac{1}{3} & -\frac{1}{3} & \frac{1}{5} & H_{22,0,-1} & 0 & 0 & 0 & 0 \\ 1 & -\frac{1}{3} & 1 & \frac{1}{5} & -\frac{1}{3} & 0 & H_{22,0,1} & 0 & 0 & 0 \\ -\frac{1}{3} & 1 & \frac{1}{5} & 1 & \frac{1}{7} & 0 & 0 & H_{22,0,-3} & 0 & 0 \\ -\frac{1}{3} & \frac{1}{5} & 1 & -\frac{1}{7} & 1 & 0 & 0 & 0 & H_{22,0,3} & 0 \\ \frac{1}{5} & -\frac{1}{3} & -\frac{1}{7} & 1 & \frac{1}{9} & 0 & 0 & 0 & 0 & H_{22,0,-5} \end{bmatrix} \begin{bmatrix} I_{1,0} \\ I_{1,-2} \\ I_{1,2} \\ I_{1,-4} \\ I_{1,4} \\ V_{2,-1} \\ V_{2,1} \\ V_{2,-3} \\ V_{2,3} \\ V_{2,-5} \end{bmatrix} \quad (22)$$

By inspection of Fig. 8 it follows that Eq. (12), which governs the voltage distribution across the diodes, will be unaffected by connecting general 2-ports (N_1 and N_2) in series with ports 1 and 2. If this is done, the 10-port can be represented as shown in Fig. 10.

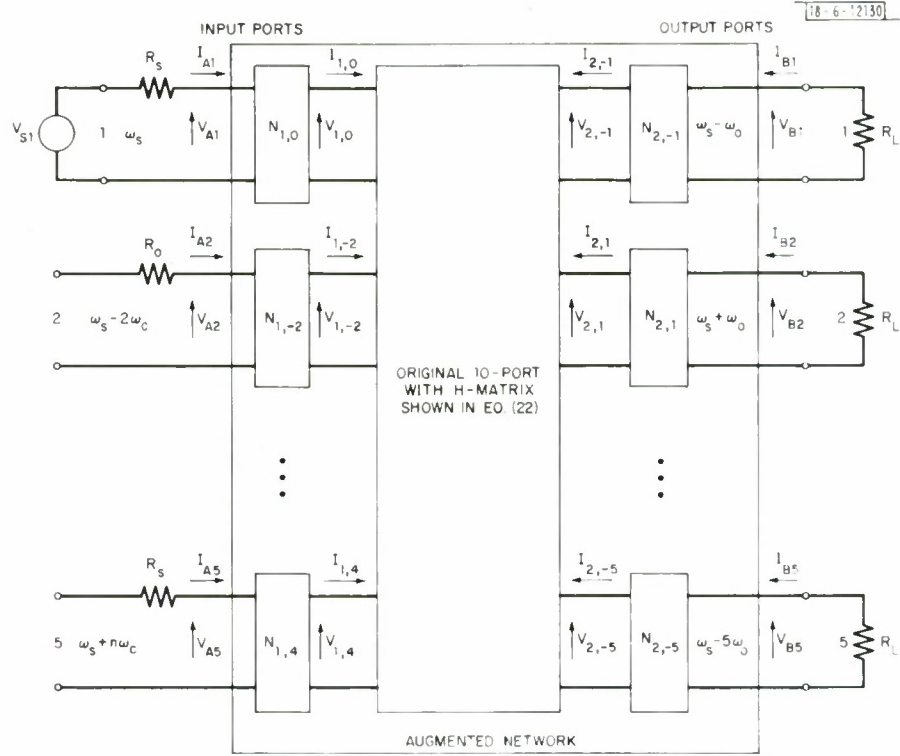


Fig. 10. Augmented 10-port network.

The networks N_1 and N_2 appear on each port evaluated at the port frequency. Let the augmented 10-port input magnitudes be denoted by \underline{V}_A and \underline{I}_A and those on the output ports by \underline{V}_B and \underline{I}_B (Fig. 10), then since the transmission matrix of $N_{1,0}$ is

$$\begin{pmatrix} V_{A1} \\ I_{A1} \end{pmatrix} = \begin{bmatrix} A_{1,0} & B_{1,0} \\ C_{1,0} & D_{1,0} \end{bmatrix} \begin{pmatrix} V_{1,0} \\ I_{1,0} \end{pmatrix}$$

the overall transmission matrix of networks $N_{1,i}(\underline{I}_1)$ becomes

$$\begin{bmatrix} \underline{V}_A \\ \underline{I}_A \end{bmatrix} = \begin{bmatrix} \underline{A}_1 & \underline{B}_1 \\ \underline{C}_1 & \underline{D}_1 \end{bmatrix} \begin{bmatrix} \underline{V}_{1,i} \\ \underline{I}_{1,i} \end{bmatrix} \quad (23)$$

where each of matrices \underline{A}_1 , \underline{B}_1 , etc., is a diagonal matrix with entries $A_{1,0}$, $A_{1,-2}$, etc., for (\underline{A}_1) in the leading diagonal. A similar matrix (\underline{T}_2) may be defined for the output ports, thus

$$\begin{bmatrix} \tilde{V}_{2,i} \\ \hline \tilde{I}_{2,i} \end{bmatrix} = \begin{bmatrix} \tilde{A}_2 & \tilde{B}_2 \\ \hline \tilde{C}_2 & \tilde{D}_2 \end{bmatrix} \begin{bmatrix} \tilde{V}_B \\ \hline \tilde{I}_B \end{bmatrix} \quad (24)$$

Converting the H-matrix of the unaugmented 10-port into an equivalent transmission matrix (T_o), then the overall transmission matrix of the augmented 10-port (T) is given by

$$[T] = [T_1] [T_o] [T_2] = \begin{bmatrix} \tilde{A} & \tilde{B} \\ \hline \tilde{C} & \tilde{D} \end{bmatrix} \quad (25)$$

If it is now assumed that the load and source resistors R_s and R_L are equal, and that $R_s R_L = 1$, then the scattering matrix parameters of the augmented network is given by

$$\tilde{S}_{11} = \tilde{U} - 2 \times (\tilde{C} + \tilde{D}) \times \Delta^{-1} \quad (26a)$$

$$\tilde{S}_{12} = \tilde{S}_{21} = -2 \times \Delta^{-1} \quad (26b)$$

$$\tilde{S}_{22} = \tilde{U} - 2 \times \Delta^{-1} \times (\tilde{B} + \tilde{D}) \quad (26c)$$

where \tilde{U} is the unit matrix and $\Delta = \tilde{A} + \tilde{B} + \tilde{C} + \tilde{D}$. The scattering matrix

$$[\tilde{S}] = \begin{bmatrix} s_{11} s_{12} & \cdot & \cdot & \cdot & s_{15} & s_{16} & \cdot & \cdot & \cdot & s_{1,10} \\ \cdot & s_{21} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \tilde{S}_{11} & \cdot & \cdot & \cdot & \tilde{S}_{12} & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ s_{51} & \cdot & \cdot & \cdot & s_{55} & s_{56} & \cdot & \cdot & \cdot & \cdot \\ \hline s_{61} & \cdot & \cdot & \cdot & s_{65} & s_{66} & \cdot & \cdot & \cdot & s_{6,10} \\ \cdot & \cdot & \tilde{S}_{21} & \cdot & \cdot & \cdot & \tilde{S}_{22} & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ s_{10,1} & \cdot & \cdot & \cdot & s_{10,5} & s_{10,6} & \cdot & \cdot & \cdot & \cdot \end{bmatrix} \quad (27)$$

has the property that

$$\begin{aligned} |s_{16}|^2 &= \frac{\text{Power delivered to output port 1}}{\text{Max. available power at input port 1}} \\ &= \frac{\text{Power delivered to frequency } (\omega_s - \omega_c)}{\text{Max. available power at source frequency } \omega_s} \end{aligned} \quad (28)$$

In similar fashion all the other $|s_{1j}|^2$ are ratios of the powers delivered to the frequency at port j to the maximum available power from the source. So an evaluation of the s_{1j} permits a

computation of how much power goes into each of the frequencies present on both sides of the mixer. Also, the input impedance Z_j at the j^{th} port:

$$Z_j = \frac{1 - s_{jj}}{1 + s_{jj}} \quad . \quad (29)$$

So from a knowledge of the s_{jj} , the input impedances on both sides of the mixer at all the frequencies are available.

Using the above mathematical background, a computer program has been written to evaluate these power and impedance distributions.

MODCAP

Based on the mathematical background described, a Fortran program was written to analyze the performance of a modulator connected between two general filters, hence the name MODCAP for MODulator Circuit Analysis Program. The MODCAP program appears at the back of this report.

The purpose of the program is to evaluate the power transfer ratio from the input port at input angular frequency ω_s to the power at all the other frequencies assumed present at both ports. For example, if the strongest five harmonics at the input and output ports are considered, the situation depicted in Fig.10 ensues. The program will evaluate the power transfer (as a ratio in dB) between input port 1 and any of the remaining nine ports. It will also evaluate the input impedances at the input and output ports at all the frequencies considered. Further, the program will evaluate the amount of change induced in the power transfer ratios and impedances if some element or elements constituting the network change by a specified amount. All of these calculations can be carried out in three different modes:

1. A single frequency is specified in the data for the input and local oscillator drive.
2. A band of input frequencies around some nominal value is specified together with a single local oscillator frequency. In this case, the output frequencies are also grouped in bands.
3. The input frequency extends over a band and a single local oscillator frequency is chosen to be a single frequency. The program will automatically change the nominal local oscillator frequency for each frequency in the input band such that a specified single output frequency results.

In each case the program evaluates the harmonics automatically at single frequencies or in frequency bands depending on the mode used. In its present form, due to storage limitations, the program has these maximum values:

1. The filters terminating the mixer on each side must have no more than 10 nodes and contain only two or three terminal elements. For ten nodes there can only be three terminal elements.
2. The maximum number of harmonics on each side of the mixer cannot exceed 10.
3. The same number of harmonics always have to be considered on each of the two ports.
4. If frequency bands around each harmonic are used, they can contain no more than 21 frequencies.
5. If effects of one or more element changes are investigated, only 42 sets of variations can be accommodated.

All of these constraints can be overridden by appropriate dimensioning.

Data is entered via a subroutine. Data for a typical case are shown in the MODCAP input data (program) at the end of this report where:

M specifies the number of frequencies in each harmonic band.

N is the number of harmonics including the fundamental considered on each side.

HINOD (a) is where a = 1 and 2 for the highest node numbers in network 1 and 2, respectively.

RISE is the fractional rise time of switching wave shape (δ_a/T) (Fig. 9c).

FALL is the fractional fall time of switching wave shape (δ_b/T) (Fig. 9e).

RT is the magnitude of source and load resistor. If unequal terminations are required transformers have to be included in networks 1 or 2.

X(a, i, j) is element X(L, R or C) of network a(1 or 2) appearing between nodes i and j. The units are Henries, Ohms and Farads and the values are entered in floating point form, e.g., $L(1, 3, 5) = 5.86E-6$ in network 1. The inductor appearing between nodes 3 and 5 has a value of $5.86 \mu H$.

If 3-terminal devices (e.g., transformers) appear in either of the terminating networks, their presence and location is described as follows:

Each device is assigned a consecutive number. Their input node is labelled E1, the output node E2 and the common node E3. The network number and device are in brackets:

$$E1(2, 1) = 1$$

$$E2(2, 1) = 2$$

$$E3(2, 1) = 3$$

This notation tells the computer that in network 2 the input node of 3-terminal device 1 is connected to the node labelled 1. The output node of device 1 in network 2 is connected to node 2 and the common node of device 1 in network 2 is connected to node 4. This describes the location of device 1 in network 2. The electrical parameters of 3-terminal devices are specified by their H-matrix. All four H-parameters must be frequency independent. H11 and H22 are in units of Ohms and Mhos, respectively; the remaining two are dimensionless. All magnitudes are again specified in floating point form. For example, $H11(2, 1) = 0.5EO$ means that H11 of device 1 in network 2 is 0.5 Ohms.

If frequency bands are used these are entered as $FR(i) = \text{value}$ where $i = 1, 2, \dots, M$. If the frequencies are equally spaced they can be entered by specifying the first frequency $FR(1) = \text{'value'}$, the increment $DR(2) = \text{'value'}$ and the last frequency $FR(7) = \text{'value'}$. Frequencies can be entered sequentially or with increments or a combination of both. There may be as many increments as necessary, but the total number of frequencies cannot exceed 24.

FLON is the nominal local oscillator frequency.

FIFN is the nominal output frequency. This is 0.0 for modes 1 and 2 and assumes some non-zero value only for mode 3.

This completes the nominal data input. If no variations are required, a RETURN card follows FIFN and then 'i CONTINUE' cards where $i = 1, 2, \dots, 42$. After 42 CONTINUE, a STOP and an END card completes the data input.

If variations are required, the 2 CONTINUE card follows the RETURN card. A 'VAR = 1' card is inserted next. Following is the first set of variations. Thus $NEWR(1, 1, 5) = 0.607EO$ means that the old value of the resistor in network 1 between nodes 1 and 5 assumes the new

value of 0.607 Ohms. At the end of the first set of variations a RETURN card has to be inserted. If there is more than one variation, the procedure is repeated with VAR = 2, etc.

NOTE:

The nodes in the two terminating networks have to be labelled as follows: the input node has to be node 1, the output node is labelled 2, internal nodes are labelled arbitrarily, and the highest node number is assigned to the common ground node.

MODCAP INPUT DATA

```

//FORG EXEC      *ORSE I-POPIEL KCC7*,MSGLEVEL=1      MODCAP
//SYSIN DD      *FORTRAN
C THIS IS THE MAIN CALLING PROGRAM MODCAP
  INTEGER HIJOB(2),VAR,LA(10),MA(10),E1(2,3),E2(2,3),E3(2,3)
  REAL    FWR(2,10,10),NEWL(2,10,10),NEWC(2,10,10),
  1L(2,10,10),R(2,10,10),C(2,10,10),8TAT(100),
  2VR(2,10,10),Y(2,10,10),F(2,10,11),FR(22),
  3DR(22),ALPHA(10,10),BETHA(10,10),QN(11,20),Q(11,20),
  4PN(11,20),P(11,20),H1(2,3),H1L(2,3),H12(2,3),H12F(2,3),
  5H21(2,3),H21F(2,3),H22(2,3),H22C(2,3)
  COMPLEX*16 AR1(100),AR2(100),BP1(100),BR2(100),AR3(100)
  COMPLEX*16 Y(2,10,10),A1(10,11),B1(10,11),C1(10,11),D1(10,11),
  1A2(10,11),B2(10,11),C2(10,11),D2(10,11),DELTA(100),
  2S11(10,10),S22(10,10),ZN(11,20),Z(11,20),DZ,SUM(20),
  3AT(10,10),T(10,10),CT(10,10),DT(10,10),
  4TAT(100),TBT(100),TCF(100),TDT(100),C(10,10),TU(100),
  5T1(100),T2(100),T11(100),T12(100),S21(10,10),Y(2,10,10)
  COMMON /4,M,MINU,RISE,FALL,VAR,FLOW,FIFN,FR,DR,NEWR,NEWL,
  INEW,C,K,L,G,(Q,P)/GL1/EL/E2,E3,H11,H1L,H12,H12F,H21,H21F,H22,H22C
  2/3L2/1,J,K,I,T,V,T,F,w/BL3/LN,PN,Z,P,Q,N,G
  DO 200 I=1,20
    SUM(I)=(0.0,0.0)
    DO 160 IMI=1,100
      AR1(IMI)=(0.0,0.0)
      AR2(IMI)=(0.0,0.0)
      AR3(IMI)=(0.0,0.0)
      BR1(IMI)=(0.0,0.0)
      BR2(IMI)=(0.0,0.0)
      BTA(IMI)=0.0
      T1(IMI)=(0.0,0.0)
      T2(IMI)=(0.0,0.0)
      TT1(IMI)=(0.0,0.0)
      TT2(IMI)=(0.0,0.0)
      TAT(IMI)=(0.0,0.0)
      TBT(IMI)=(0.0,0.0)
      TCF(IMI)=(0.0,0.0)
      TDT(IMI)=(0.0,0.0)
    200
  160
  10000010
  10000020
  10000030
  10000040
  10000050
  10000060
  10000070
  10000080
  10000090
  10000100
  10000110
  10000120
  10000130
  10000140
  10000150
  10000160
  10000170
  10000180
  10000190
  10000200
  10000210
  10000220
  10000230
  10000240
  10000250
  10000260
  10000270
  10000280
  10000290
  10000300
  10000310
  10000320
  10000330
  10000340
  10000350

```

```

160      DELTA((,I))=(0.,0.,0.)
      DO 170 I=1,10
      DO 170 J=1,10
      AT(I,J)=(0.,0.,0.)
      BT(I,J)=(0.,0.,0.)
      CT(I,J)=(0.,0.,0.)
      DT(I,J)=(0.,0.,0.)
      S1(I,J)=(0.,0.,0.)
      S2(I,J)=(0.,0.,0.)
      S21(I,J)=(0.,0.,0.)
      ALPHA((,J))=0.0
      BETA((,J))=0.0
      DO 150 I = 1,2
      DO 150 J=1,10
      DO 150 K=1,10
      YI(I,J,K)=(0.,0.,0.)
      RI(I,J,K) = 0.0
      LI(I,J,K) = 0.0
      CI(I,J,K) = 0.0
      Y4(I,J,K)=0.0
      YI((,J,K))=0.0
      NEWR((,J,K))=0.0
      NEWL(I,J,K)=0.0
      NEWC(I,J,K)=0.0
      Y(I,J,K)=(0.,0.,0.)
      DO 151 I=1,22
      DR(I) = 0.0
      FR(I) = 0.0
      DO 180 I=1,11
      DO 180 J=1,20
      ZN((,J))=(0.,0.,0.)
      Z(I,J)=(0.,0.,0.)
      ZN(I,J)=0.0
      Q(I,J)=0.0
      PN((,J))=0.0
150
151

```

M0000360
M0000370
M0000380
M0000390
M0000400
M0000410
M0000420
M0000430
M0000440
M0000450
M0000460
M0000470
M0000480
M0000490
M0000500
M0000510
M0000520
M0000530
M0000540
M0000550
M0000560
M0000570
M0000580
M0000590
M0000600
M0000610
M0000620
M0000630
M0000640
M0000650
M0000660
M0000670
M0000680
M0000690
M0000700


```

180      P(I,J)=0.0
      DO 190 I=1,10
      DO 190 J=1,11
      A1(I,J)=(0.0,0.0)
      B1(I,J)=(0.0,0.0)
      C1(I,J)=(0.0,0.0)
      D1(I,J)=(0.0,0.0)
      A2(I,J)=(0.0,0.0)
      B2(I,J)=(0.0,0.0)
      C2(I,J)=(0.0,0.0)
      D2(I,J)=(0.0,0.0)
      DO 310 I=1,2
      DO 310 J=1,3
      H11(I,J)=0.0
      H11L(I,J)=0.0
      H12(I,J)=0.0
      H12F(I,J)=0.0
      H21(I,J)=0.0
      H21F(I,J)=0.0
      H22(I,J)=0.0
      H22C(I,J)=0.0
      E1(I,J)=0.0
      E2(I,J)=0.0
      E3(I,J)=0.0
      VAR = 0
      ISQ=0
      IQ = 1
      CALL DATA
      DO 1 I=1,2
      J = H10D(I)
      DO 1 IX=1,J
      DO 1 IY=1,J
      NEWA(I,IX,IY) = P(I,IX,IY)
      NEWB(I,IX,IY) = L(I,IX,IY)
      NEWC(I,IX,IY) = C(I,IX,IY)
      1 CONTINUE
270 CONTINUE
      IF (IQ-1) 3,2,1
      3 BEGINNING OF SUBROUTINE ALPHA
      2 G1 = RISE
      52 = FALL
      MU000710
      MU000720
      MU000730
      MU000740
      MU000750
      MU000760
      MU000770
      MU000780
      MU000790
      MU000800
      MU000810
      MU000820
      MU000830
      MU000840
      MU000850
      MU000860
      MU000870
      MU000880
      MU000890
      MU000900
      MU000910
      MU000920
      MU000930
      MU000940
      MU001000
      MU001010
      MU001020
      MU001030
      MU001040
      MU001050
      MU001060
      MU001070
      MU001080
      MU001090
      MU001100
      MU001110
      MU001120
      MU001130
      MU001140
      MU001150

```

```

120      PI = 3.141593
121      IE = 1
122      JF = 1
123      M1 = (-1)**JF
124      M2 = (-1)**IE
125      Q1 = IE
126      QJ = JF
127      IF (M1-G2) 122,123,124
128      ALPHA(IE,JF) = (-1.)*((JF-IE+1)/2)/(0.5*PI*(Q1-QJ))
129      GO TO 134
130      ALPHA(IE,JF) = (-1.)*((IE+JF-2)/2)/(0.5*PI*(Q1+QJ-1))
131      GO TO 134
132      ALPHA(IE,JF) = (-1.)*((IL-JF+1)/2)/(0.5*PI*(QJ-Q1))
133      IF (G1+G2) 125,126,125
134      Z1 = 1/ABS(ALPHA(IE,JF))
135      IF (G1) 128,127,128
136      X1 = 0.5
137      GO TO 130
138      X1 = (SIN(Z1*PI*G1))/(2.*Z1*PI*G1)
139      IF (G2) 130,129,130
140      YJ = 0.5
141      GO TO 131
142      YJ = (SIN(Z1*PI*G2))/(2.*Z1*PI*G2)
143      ALPHA(IE,JF) = (X1+YJ)*ALPHA(IL,JF)
144      IE = IL+1
145      IF (IE-N) 121,121,132
146      JF = JF+1
147      IE = I
148      IF (JF-N) 120,120,133
149      C END OF SUBROUTINE ALPHA
150      CALL ARRAY(2,N,N,IO,IO,BIA,ALPHA)
151      CALL M1VCTA,N,DET,LA,MA)
152      CALL ARRAY(1,N,N,IO,IO,ETA,BEITA)
153      C THE FREQUENCY EVALUATION CARDS BEGIN HERE. IF USED AS A SUBROUTINE
154      C THIS PART IS CALLED 'SUBROUTINE FREQ'

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M0001160
M0001170
M0001180
M0001190
M0001200
M0001210
M0001220
M0001230
M0001240
M0001250
M0001260
M0001270
M0001280
M0001290
M0001300
M0001310
M0001320
M0001330
M0001340
M0001350
M0001360
M0001370
M0001380
M0001390
M0001400
M0001410
M0001420
M0001430
M0001440
M0001450
M0001460
M0001470
M0001480
M0001490
M0001500

```

51	I = 0	M0001510
	J = 0	M0001520
	J = J+1	M0001530
52	IF(FR(J)) 51,52,51	M0001540
53	IF(J-N) 53,53,57	M0001550
54	I1 = J-1	M0001560
	J = J+1	M0001570
	IF(FR(J)) 55,54,55	M0001580
55	I = I1	M0001590
56	I = I+1	M0001600
	FR(I) = FR(I-1)+DR(I1)	M0001610
	IF(I-J+1) 56,51,56	M0001620
57	J = 1	M0001630
58	K = 1	M0001640
	MAA = (-1)*J	M0001650
	IF(MAA) 64,64,59	M0001660
59	IF(K-M) 60,60,69	M0001670
60	IF(FIFN) 62,61,62	M0001680
61	F(1,J,K) = ABS(FR(K)-J*FLON)	M0001690
	F(2,J,K) = FR(N)+(J-1)*FLON	M0001700
	GO TO 63	M0001710
62	F(1,J,K) = ABS(FR(K)-J*(FR(K)-FIFN))	M0001720
	F(2,J,K) = FIFN+J*FLON	M0001730
63	K = K+1	M0001740
	GO TO 59	M0001750
64	IF(K-M) 65,65,69	M0001760
65	IF(FIFN) 67,66,67	M0001770
66	F(1,J,K) = FR(N)+(J-1)*FLON	M0001780
	F(2,J,K) = ABS(FR(K)-J*FLON)	M0001790
	GO TO 68	M0001800
67	F(1,J,K) = FR(K)+(J-1)*(FR(K)-FIFN)	M0001810
	F(2,J,K) = ABS(FIFN-(J-1)*FLON)	M0001820
68	K = K+1	M0001830
	GO TO 64	M0001840
69	J = J+1	M0001850
	IF(J-N) 58,58,6	M0001860
C THE	FREQUENCY EVALUATION CARDS END HERE	M0001870
4	GO TO 70	M0001880
3	CONTINUE	

```

C THE FILTER EVALUATION CARDS BEGIN HERE. IF USED AS A SUBROUTINE
C THIS PART IS CALLED "SUBROUTINE DMATE" (FOR DIAGONAL MATRIX
C EVALUATION)
70 I = 1
71 J = 1
72 K = 1
73 IX = 1
74 IY = 1
94 IF(NEWC(I,IX,IY)) 72,73,72
72 YR(I,IX,IY) = -RT/NEWC(I,IX,IY)
80 TO 74
73 YR(I,IX,IY) = C.0
74 W = 2*3.141592654*(I,J,K)
75 IF(K-1) 75,77,75
75 IF(F(I,J,1)-F(I,J,K)) 77,76,77
76 A2(J,K) = A2(J,1)
77 B2(J,K) = B2(J,1)
78 C2(J,K) = C2(J,1)
79 D2(J,K) = D2(J,1)
80 TO 87
77 IF(NEWC(I,IX,IY)) 77,78,79
78 YI(I,IX,IY) = -W*NEWC(I,IX,IY)*RT
80 TO 80
79 YI(I,IX,IY) = (RT/(W*NEWC(I,IX,IY)))-(W*NEWC(I,IX,IY)*RT)
80 A = YR(I,IX,IY)
81 B = YI(I,IX,IY)
82 Y(I,IX,IY) = COMPLEX(A,B)
83 IX = IX+1
84 IF(IX-HINDO(I)) 99,99,81
85 IX = 1
86 IY = IY+1
87 IF (IY-HINDO(I)) 99,99,82
88 IS = HINDO(I)
89 DO 152 IX = 2,IS
90 KI = IX-1
91 DO 152 IY=1,KI
92 Y(I,IX,IY) = Y(I,IX,IY)+Y(I,IX,IY)
93 Y(I,IY,IX) = Y(I,IX,IY)
94 IA = HINDO(I)
95 JA=I+IA

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M0001930
M0001940
M0001950
M0001960
M0001970
M0001980
M0001990
M0002000
M0002010
M0002020
M0002030
M0002040
M0002050
M0002060
M0002070
M0002080
M0002090
M0002100
M0002110
M0002120
M0002130
M0002140
M0002150
M0002160
M0002170
M0002180
M0002190
M0002200
M0002210
M0002220
M0002230
M0002240
M0002250
M0002260
M0002270
M0002280
M0002290
M0002300
M0002310
M0002320

91	SUM(JA) = (0.0,0.0)	MDD02330
92	DO 91 JB=1,IA	MDD02340
	SUM(JA) = SUM(IA) + Y(1,JA,JB)	MDD02350
	Y(1,JA,JA) = -SUM(JA)	MDD02360
311	DO 313 JT=1,3	MDD02370
	IF(EI(1,JT)) 313,313,311	MDD02380
	CALL TKOPRT	MDD02390
	DO 312 JA=1,IA	MDD02400
312	DO 312 JB=1,IA	MDD02410
313	Y(1,JA,JB) = Y(1,JA,JB) + Y(1,JA,JB)	MDD02420
	CONTINUE	MDD02430
83	IF(HINDD(1)-3) 184,184,83	MDD02440
	IA = HINDD(1)-3	MDD02450
	DO 84 IZ=1,IA	MDD02460
	IS = HINDD(1)-IZ	MDD02470
	IB = IS-I	MDD02480
	DO 84 IY=1,IB	MDD02490
	DO 84 IX=1,IB	MDD02500
84	Y(1,IX,IY) = Y(1,IX,IY) - (Y(1,IX,IS)*Y(1,IS,IY)/Y(1,IS,IS))	MDD02510
184	CONTINUE	MDD02520
85	IF(I-1) 85,85,86	MDD02530
	A1(J,K) = -Y(1,2,2)/Y(1,2,1)	MDD02540
	B1(J,K) = -1.0/Y(1,2,1)	MDD02550
	C1(J,K) = -(Y(1,1,1)*Y(1,2,2)-Y(1,1,2)*Y(1,2,1))/Y(1,2,1)	MDD02560
	D1(J,K) = -Y(1,1,1)/Y(1,2,1)	MDD02570
	GO TO 87	MDD02580
86	A2(J,K) = -Y(2,2,2)/Y(2,2,1)	MDD02590
	B2(J,K) = -1.0/Y(2,2,1)	MDD02600
	C2(J,K) = -(Y(1,1,1)*Y(2,2,2)-Y(2,1,2)*Y(2,2,1))/Y(2,2,1)	MDD02610
	D2(J,K) = -Y(2,1,1)/Y(2,2,1)	MDD02620
87	K = K+1	MDD02630
	IF(K-M) 88,88,89	MDD02640
88	IX = 1	MDD02650
	IY = 1	MDD02660
89	GO TO 99	MDD02670
	K = 1	MDD02680
	J = J+1	MDD02690
	IF(J-N) 88,88,90	MDD02700
90	I = I+1	MDD02710
	IF((-2) 71,71,95	MDD02720

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C END OF FILTER EVALUATION CARDS
95 DD 18 K=L,M
DD 5 IX=L,N
DD 5 IY=L,N
AT(IX,IY)=A1(IY,K)*ALPHA(IX,IY)*A2(IY,K)+B1(IX,K)*BETHA(IX,IY)
I*C2(IY,K)
BT(IX,IY)=A1(IY,K)*ALPHA(IX,IY)*B2(IY,K)+B1(IX,K)*BETHA(IX,IY)
I*D2(IY,K)
CT(IX,IY)=C1(IY,K)*ALPHA(IX,IY)*A2(IY,K)+D1(IX,K)*BETHA(IX,IY)
I*C2(IY,K)
DT(IX,IY)=C1(IY,K)*ALPHA(IX,IY)*B2(IY,K)+D1(IX,K)*BETHA(IX,IY)
I*D2(IY,K)
C BEGINNING OF MATRIX CALCULATIONS
DD 250 IO=L,N
DD 250 JO=L,N
IF(IO-JO) 202,201,202
201 J(IG,IJ)=(1.0,C.0)
GD TO 250
202 J(IO,JO)=(0.0,C.0)
250 CONTINUE
CALL CARRAY (2,N,N,10,10,IAT,AT)
CALL CARRAY (2,N,N,10,10,IBT,BT)
CALL CARRAY (2,N,N,10,10,ICT,CT)
CALL CARRAY (2,N,N,10,10,IUT,UT)
CALL CARRAY (2,N,N,10,10,IU,U)
CALL CGMADD (IAT,IBT,AK1,N,N)
CALL CGMADD (ICT,IUT,AK2,N,N)
CALL CGMADD (IUT,IUT,AK3,N,N)
CALL CGMADD (AK1,AK2,DELTA,N,N)
CALL CMINV (DELTA,N,DEI,LA,MA)
KD=N*N
DD 203 IP=L,KJ
DELTA(IP)=2.0*DELTA(IP)
203 DD 161 IN=L,N
CALL LOC (IN,L,IJ,N,N,0)
S21(IN,I)=-1.0*DELTA(IJ)
161 CALL CGMPRO(AR2,DELTA,IJ,N,N,N)
CALL CGMSUB (IIP,IJ,IJL,N,N)
DD 96 IM=L,N
CALL LOC (IM,L,IJ,N,N,0)
S11(IM,I)=I11(IJ)
CALL LOC (IN,I",IJ,N,N,0)
M0002730
M0002740
M0002750
M0002760
M0002770
M0002780
M0002790
M0002800
M0002810
M0002820
M0002830
M0002840
M0002850
M0002860
M0002870
M0002880
M0002890
M0002900
M0002910
M0002920
M0002930
M0002940
M0002950
M0002960
M0002970
M0002980
M0002990
M0003000
M0003010
M0003020
M0003030
M0003040
M0003050
M0003060
M0003070
M0003080
M0003090
M0003100
M0003110
M0003120
M0003130
M0003140

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96      S11(IM,IM)=T11(IJ)
      CALL CG4PRD(DELTA,K3,I2,N,N,N)
      CALL COMSUB (T1,T2,T2,N,N)
      DO 97 I4=1,N
      CALL LDC(IN,I4,I4,N,0)
97      S22(IN,IN) = T2(IJ)
      C END OF MATRIX CALCULATIONS
      DO 18 IX=1,N
      IF(I0-1) 12,6,12
      PN(K,IX) = REAL(S11(IX,1)*DCONJG(S11(IX,1)))
      PN(K,IX) = 10.*ALOG10(PN(K,IX))
      PN(K,(IX+N)) = REAL(S21(IX,1)*DCONJG(S21(IX,1)))
      PN(K,(IX+N)) = 10.*ALOG10(PN(K,(IX+N)))
      Y1 = AIMAG(S11(IX,IX))
      IF(Y1) 9,46,8
      X1 = REAL(S11(IX,IX))
      IF(X1+1) 8,7,8
      QN(K,IX) = 0.0
      GO TO 9
      6      QN(K,IX) = 1.0
      ZN(K,IX) = -((S11(IX,IX)-1.0)/(S11(IX,IX)+1.0))*RT
      Y2 = AIMAG(S22(IX,IX))
      IF(Y2) 11,47,11
      X2 = REAL(S22(IX,IX))
      IF(X2+1) 11,10,11
      QN(K,(IX+N)) = 0.0
      GO TO 18
      11      QN(K,(IX+N)) = 1.0
      ZN(K,(IX+N)) = -((S22(IX,IX)-1.0)/(S22(IX,IX)+1.0))*RT
      GO TO 18
      12      P(K,IX) = REAL(S11(IX,1)*DCONJG(S11(IX,1)))
      P(K,IX) = 10.*ALOG10(P(K,IX))
      P(K,(IX+N)) = REAL(S21(IX,1)*DCONJG(S21(IX,1)))
      P(K,(IX+N)) = 10.*ALOG10(P(K,(IX+N)))
      Y1 = AIMAG(S11(IX,IX))
      IF(Y1) 14,48,14
      X1 = REAL(S11(IX,IX))
      IF(X1+1) 14,13,14
      Q(K,IX) = 0.0
      GO TO 15

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M0003150
M0003160
M0003170
M0003180
M0003190
M0003200
M0003210
M0003220
M0003230
M0003240
M0003250
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M0003270
M0003280
M0003290
M0003300
M0003310
M0003320
M0003330
M0003340
M0003350
M0003360
M0003370
M0003380
M0003390
M0003400
M0003410
M0003420
M0003430
M0003440
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M0003460
M0003470
M0003480
M0003490
M0003500
M0003510
M0003520
M0003530
M0003540

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14      Q(K,IX) = 1.0
15      Z(K,IX) = -((S11(IX,IX)-1.0)/(S11(IX,IX)+1.0))*RT
      Y2 = ALMAG(S22(IX,IX))
      IF(Y2) 17,49,17
43      X2 = XLAL(S22(IX,IX))
      IF(X2+1) 17,16,17
16      Q(K,(IX+N)) = 0.0
      GO TO 13
17      Q(K,(IX+N)) = 1.0
      Z(K,(IX+N)) = -((S22(IX,IX)-1.0)/(S22(IX,IX)+1.0))*RT
18      CONTINUE
504     WRITE (6,504) 'Q'
      FORMAT (F5,'IQ=',I2)
      CALL PRINT
314     IF(ISO) 311,314,311
      CONTINUE
      IQ = IQ+1
260     IF(IG-2) 262,262,260
      DO 261 I=1,2
      J = MINOD(I)
      DO 261 IX = 1,1
      DO 261 IY = 1,1
      NEWR(1,IX,IY) = R(1,IX,IY)
      NEWL(1,IX,IY) = L(1,IX,IY)
      NEWC(1,IX,IY) = C(1,IX,IY)
      CONTINUE
261     CALL DATA
262     IF(ISO) 311,314,311
315     CONTINUE
      GO TO 270
END
SUBROUTINE DATA
INTEGER MINOD(2),VAR,E1(2,3),E2(2,3),E3(2,3)
REAL R(2,10,10),L(2,10,10),C(2,10,10),FR(22),DR(22),
1NEWR(2,10,10),LEWL(2,10,10),NEWC(2,10,10),H1L(2,3),H1LL(2,3),
2H12(2,3),H12F(2,3),H21(2,3),H21F(2,3),H22(2,3),H22C(2,3)
COMMON '1,M',MINOD,RISE,FALL,VAR,FLON,FIFN,FR,DR,
1NEWR,NEWL,NEWC,K,L,C,IQ,RT/BL1/CL,E2,E3,
2H1L,H1LL,H12,H12F,H21,H21F,H22,H22C
      GO TO (1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,
123,24,25,26,27,28,29,30,31,32,33,34,35,36,37,38,39,40,41,42),IQ

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M0003550
M0003560
M0003570
M0003580
M0003590
M0003600
M0003610
M0003620
M0003630
M0003640
M0003650

M0004110
M0004120
M0004130
M0004140
M0004150
M0004160
M0004170
M0004180
M0004190
M0004200
M0004210

M0004370
M0004380
M0004390
M0004400
M0004410
M0004420
M0004430
M0004440
M0004450
M0004460
M0004470
M0004480

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1      CONTINUE
      M=7
      N=5
      HINUD(1) = 6
      HINUD(2)=4
      RISE = 0.1
      FALL = 0.1
      RT = 50.0
      L(1,3,5)=5.68E-6
      C(1,3,4)=90.94E-12
      C(1,2,4)=2.0E-12
      R(1,2,4)=5.0E0
      R(1,1,5)=2.5E0
      R(2,1,3)=1.2E0
      R(2,1,4)=2.46E3
      R(2,3,4)=1.2E6
      L(2,1,4)=1.96E-6
      C(2,1,4)=3.225E-9
      C(2,3,4)=8.0E-12
      E1(2,1)=1
      E2(2,1)=2
      E3(2,1)=4
      H11(2,1)=0.5E0
      H12(2,1)=1.570796E0
      H21(2,1)=-1.570796E0
      H22(2,1)=1.25E-3
      FR(1)=6.7E6
      OR(1)=0.1E6
      FR(7)=7.3E6
      FLUN = 5.0E6
      FIFN = 0.0
      RETURN

2      CONTINUE
      VAK=1
      NEWR(1,1,5)=0.407E0
      NEWC(1,3,4)=3.732E-10
      NEWL(1,3,5)=1.384E-6
      NEWL(2,1,4)=9.82E-6
      NEWC(2,1,4)=6.45E-10
      NEWR(2,1,4)=12.3E3
      RETURN
M0004490
M0004510
M0004520
M0004530
M0004540
M0004550
M0004560
M0004570
M0004580
M0004590
M0004600
M0004610
M0004620
M0004630
M0004640
M0004650
M0004660
M0004670
M0004680
M0004690
M0004700
M0004720
M0004730
M0004740
M0004760
M0004780
M0004790
M0004800
M0004820
M0004830
M0004840
M0004850
M0004860
M0004870
M0004880
M0004890
M0004900

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M0004910
M0004920
M0004930
M0004940
M0004950
M0004960
M0004970
M0004980
M0004990
M0005000

3 CONTINUE
VAR=2
NEWB(1,1,5)=0.607E0
NEWC(1,3,4)=3.732E-10
NEWL(1,3,5)=1.384E-6
NEWL(2,1,4)=3.27E-6
NEWB(2,1,4)=1.035E-9
NEWB(2,1,4)=4.1E3
RETURN
4 CONTINUE
5 CONTINUE
6 CONTINUE
7 CONTINUE
8 CONTINUE
9 CONTINUE
10 CONTINUE
11 CONTINUE
12 CONTINUE
13 CONTINUE
14 CONTINUE
15 CONTINUE
16 CONTINUE
17 CONTINUE
18 CONTINUE
19 CONTINUE
20 CONTINUE
21 CONTINUE
22 CONTINUE
23 CONTINUE
24 CONTINUE
25 CONTINUE
26 CONTINUE
27 CONTINUE
28 CONTINUE
29 CONTINUE
30 CONTINUE
31 CONTINUE
32 CONTINUE
33 CONTINUE

34	CONTINUE	
35	CONTINUE	
36	CONTINUE	
37	CONTINUE	
38	CONTINUE	
39	CONTINUE	
40	CONTINUE	
41	CONTINUE	
42	STOP	
	END	
C	MOD04930
C		MOD08430
C		MOD05240
C		MOD05250
C	SUBROUTINE CMINV	MOD05260
C	PURPOSE	MOD05270
C		MOD05280
C		MOD05290
C		MOD05300
C	INVERT A GENERAL MATRIX	MOD05310
C	CONTAINING COMPLEX ELEMENTS	MOD05320
C		MOD05330
C	USAGE AND CALLING CONVENTION	MOD05340
C	SAME AS FOR MINV.	MOD05350
C		MOD05360
	SUBROUTINE CMINV(A,N,D,L,M)	MOD05370
	DIMENSION L(1),M(1)	MOD05380
	COMPLEX*16 A(1),D,HIGA,HULD	MOD05390
	D=(1.0,0.0)	MOD05400
	NK=-1	MOD05410
	DO 80 K=1,N	MOD05420
	NK=NK+N	MOD05430
	L(K)=K	MOD05440
	M(K)=K	MOD05450
	KK=NK+K	MOD05460
	HIGA=A(KK)	MOD05470
	DO 20 J=K,N	MOD05480
	I2 = N*(J-1)	MOD05490
	DO 20 I=K,J	MOD05500
	IJ=I2+I	MOD05510
10	IF(CDABS(HIGA)-CDABS(A(IJ))) 15,20,20	MOD05520
15	HIGA=A(IJ)	MOD05530
	L(K)=I	MOD05540

20	M(K)=J CONTINUE J=L(K) IF(J-K) 35,35,75	M000555D M000556D M000557D M000558D
25	KI=K-N DO 30 I=1,4 KI=KI+N HOLD=-A(KI) JI=KI-K+J A(KI)=A(JI) A(JI)=HOLD I=M(K) IF(I-K) 45,45,78	M000559D M000560D M000561D M000562D M000563D M000564D M000565D M000566D M000567D M000568D
30	JP=N*(I-1)	M000569D
35	DO 40 J=1,4 JK=NK+J JI=JP+J HOLD=-A(JK) A(JK)=A(JI) A(JI)=HOLD IF(CDACS(BIGA)) 48,46,48	M000570D M000571D M000572D M000573D M000574D M000575D
40	D=(0.0,0.0)	M000576D
45	RETURN	M000577D
46	DO 55 I=1,4 IF(I-K) 50,55,50 IK=NK+I A(IK)=A(IK)/(-PIGA)	M000578D M000579D M000580D M000581D
50	CONTINUE	M000582D
55	DO 65 I=1,4 IK=NK+I HOLD=A(IK) IJ=I-N DO 65 J=1,4 IJ=IJ+N IF(I-K) 60,65,60 IF(J-K) 62,65,72 KJ=(J-(*K A(IJ)=HOLD*A(KI)+A(IJ)	M000583D M000584D M000585D M000586D M000587D M000588D M000589D M000590D M000591D M000592D
60		
62		

65	CONTINUE	M0005930
	KJ=K-N	M0005940
	DO 75 J=I,N	M0005950
	KJ=KJ+N	M0005960
70	IF(J-K) 70,75,70	M0005970
75	A(KJ)=A(KJ)/B1/A	M0005980
	CONTINUE	M0005990
	D=D*B1/A	M0006000
80	A(KK)=1.0/B1/A	M0006010
	CONTINUE	M0006020
	K=N	M0006030
100	K=(K-1)	M0006040
	IF(K) 150,150,105	M0006050
105	I=L(K)	M0006060
	IF(I-K) 120,120,108	M0006070
108	JQ=N*(K-1)	M0006080
	JR=N*(I-1)	M0006090
	DO 110 J=1,N	M0006100
	JK=JQ+J	M0006110
	HOLD=A(JK)	M0006120
	J1=JR+J	M0006130
	A(JK)=-A(J1)	M0006140
110	A(J1)=HOLD	M0006150
120	J=M(K)	M0006160
	IF(J-K) 100,100,125	M0006170
125	KI=K-N	M0006180
	DO 130 I=1,N	M0006190
	KI=KI+I	M0006200
	HOLD=A(KI)	M0006210
	J1=KI-K+J	M0006220
	A(KI)=-A(J1)	M0006230
130	A(J1)=HOLD	M0006240
	GO TO 100	M0006250
150	RETURN	M0006260
	END	M0006270


```

SUBROUTINE CARPAY (MODE,I,J,N,M,S,D)
COMPLEX*16 S(1),D(1)
NI = N-I
IF(MODE-1) 100, 100, 120
IJ=I+J+1
NM=N+J+1
DO 110 K=1,J
NM=NM-NI
DO 110 L=1,I
IJ=IJ-1
NM=NM-1
D(NM)=S(IJ)
GO TO 140
IJ=0
NM=0
DO 130 K=1,J
DO 125 L=1,I
IJ=IJ+1
NM=NM+1
S(IJ)=D(NM)
NM=NM+NI
RETURN
END
SUBROUTINE ARRAY (MODE,I,J,N,M,S,D)
DIMENSION S(1),D(1)
NI = N-I
IF(MODE-1) 100, 100, 120
IJ=I+J+1
NM=N+J+1
DO 110 K=1,J
NM=NM-NI
DO 110 L=1,I
IJ=IJ-1
NM=NM-1
D(NM)=S(IJ)
GO TO 140
IJ=0
NM=0
DO 130 K=1,J
DO 125 L=1,I
IJ=IJ+1
NM=NM+1

```

100

110

120

125

130

140

100

110

120

125	S(IJ)=U(NM)	M0007530
130	NM=NM+1	M0007540
140	RETURN	M0007550
	END	M0007560
	SUBROUTINE LOC(I,J,IR,N,M,N,S)	M0007570
	IX = I	M0007580
	JX = J	M0007590
	IF (MS-1) 10,20,30	M0007600
10	IRX = N*(JX-1)+IX	M0007610
	GO TO 36	M0007620
20	IF (IX-JX) 22,24,24	M0007630
22	IRX = IX+(JX*JN-JX)/2	M0007640
	GO TO 36	M0007650
24	IRX = JX+(IX*IX-IX)/2	M0007660
	GO TO 36	M0007670
30	IRX = 0	M0007680
	IF (IX-JX) 36,32,36	M0007690
32	IRX = IX	M0007700
36	IR = IRX	M0007710
	RETURN	M0007720
	END	M0007730
	SUBROUTINE PLIV(A,N,D,L,M)	M0007740
	DIMENSION A(1),L(1),M(1)	M0007750
	D = 1.0	M0007760
	NK=-N	M0007770
	DO 60 K=1,N	M0007780
	NK=NK+N	M0007790
	L(K)=K	M0007800
	M(K)=K	M0007810
	KK=K+K	M0007820
	BIGA=A(KK)	M0007830
	DO 20 J=K,N	M0007840
	I2 = N*(J-1)	M0007850
	DO 20 I=K,N	M0007860
	IJ=I2+I	M0007870
10	IF(ABS(BIGA)-A(IJ)) 15,20,20	M0007880
15	BIGA=A(IJ)	M0007890
	L(K)=I	M0007900
	M(K)=J	M0007910

20	CONTINUE	M0007920
	J=L(K)	M0007930
25	IF(J-K) 35,35,75	M0007940
	KI=K-N	M0007950
	DO 30 I=1,N	M0007960
	KI=KI+N	M0007970
	HOLD=-A(KI)	M0007980
	JI=KI-K+J	M0007990
30	A(KI)=A(JI)	M0008000
35	A(JI)=HOLD	M0008010
	I=M(K)	M0008020
	IF(I-K) 45,45,78	M0008030
38	JP=N*(I-1)	M0008040
	DO 40 J=1,N	M0008050
	JK=NK+J	M0008060
	JJ=JP+J	M0008070
	HOLD=-A(JK)	M0008080
	A(JK)=A(JI)	M0008090
40	A(JI)=HOLD	M0008100
45	IF(BIGA) 48,46,48	M0008110
46	D = 0.0	M0008120
	RETURN	M0008130
48	DO 55 I=1,N	M0008140
	IF(I-K) 50,55,50	M0008150
50	IK=IK+I	M0008160
	A(IK)=A(IK)/(-BIGA)	M0008170
55	CONTINUE	M0008180
	DO 65 I=1,N	M0008190
	IK=NK+I	M0008200
	HOLD=A(IK)	M0008210
	IJ=I-N	M0008220
	DO 65 J=1,N	M0008230
	IJ=IJ+N	M0008240
	IF(I-K) 60,65,70	M0008250
60	IF(J-K) 62,65,72	M0008260
62	KJ=IJ-I+K	M0008270
	A(IJ)=HOLD+A(KI)+A(IJ)	M0008280
65	CONTINUE	M0008290
	KJ=K-N	M0008300
	DO 75 J=1,N	M0008310
	KJ=KJ+N	M0008320
	IF(J-K) 70,75,70	M0008330

70	A(KJ)=A(KJ)/B1*A	M0008340
75	CONTINUE	M0008350
	D=0*B1GA	M0008360
	A(KK)=1.0/B1GA	M0008370
80	CONTINUE	M0008380
	K=N	M0008390
100	K=(K-1)	M0008400
	IF(K) 150,150,105	M0008410
105	I=L(K)	M0008420
	IF(I-K) 120,120,108	M0008430
108	JQ=N*(K-1)	M0008440
	JR=N*(I-1)	M0008450
	DO 110 J=1,N	M0008460
	JK=JQ+J	M0008470
	HOLD=A(JK)	M0008480
	J1=JR+J	M0008490
	A(JK)=-A(J1)	M0008500
110	A(J1)=HOLD	M0008510
120	J=M(K)	M0008520
	IF(J-K) 100,100,125	M0008530
125	K1=K-N	M0008540
	DO 130 I=1,N	M0008550
	KI=K1+N	M0008560
	HOLD=A(KI)	M0008570
	J1=KI-K+J	M0008580
	A(KI)=-A(J1)	M0008590
130	A(J1)=HOLD	M0008600
150	GO TO 100	M0008610
	RETURN	M0008620
	END	M0008630
C	M0008640
C	SUBROUTINE TWOPT	M0008650
C	M0008660
C	PURPOSE	M0008670
C		M0008680
C	TO CONVERT THE TWO-PORT H MATRIX PARAMETERS SPECIFIED IN THE	M0008690
C	DATA SUBROUTINE , INTO Y MATRIX PARAMETERS AND TO ASSIGN TO	M0008700
C	THEM THEIR PROPER LOCATIONS IN THE OVERALL INDEFINITE	M0008710
C	ADMITTANCE MATRIX	M0008720

C	MOD08730
	SUBROUTINE TWOPRT	MOD08740
	INTEGER E1(2,3),E2(2,3),E3(2,3)	MOD08750
	REAL H11(2,3),H11L(2,3),H12(2,3),H12F(2,3),H21(2,3),	MOD08760
	H21F(2,3),H22(2,3),H22C(2,3),F(2,10,11),A(4),B(4)	MOD08770
	COMPLEX*16 Y1(2,10,10),DEL,X1,X2,X3,X4	MOD08780
	COMMON /BL1/ E1,E2,E3,H11,H11L,H12,H12F,H21,H21F,H22,H22C	MOD08790
	I/BL2/ I,J,K,JT,YT,I,W	MOD08800
	DO 10 J1=1,2	MOD08810
	DO 10 J2=1,10	MOD08820
	DO 10 J3=1,10	MOD08830
10	YT(J1,J2,J3)=(0.0,0.0)	MOD08840
	DO 11 I1=1,4	MOD08850
	A(I1)=0.0	MOD08860
11	B(I1)=0.0	MOD08870
	A(1)=H11(,JT)	MOD08880
	B(1)=W*H11L(1,JT)	MOD08890
	X1=CMPLX(A(1),B(1))	MOD08900
	A(2)=H22(1,JT)	MOD08910
	B(2)=W*H22C(1,JT)	MOD08920
	X2=CMPLX(A(2),B(2))	MOD08930
	IF(H12F(1,JT)) 1,2,1	MOD08940
1	B(3)=F(1,J,K)/H12F(1,JT)	MOD08950
	X3=CMPLX(1.,B(3))	MOD08960
	X3=H12(1,JT)/X3	MOD08970
	GO TO 3	MOD08980
2	A(3)=H12(1,JT)	MOD08990
	X3=CMPLX(A(3),0.0)	MOD09000
3	IF(H21F(1,JT)) 4,5,4	MOD09010
4	B(4)=F(1,J,K)/H21F(1,JT)	MOD09020
	X4=CMPLX(1.,B(4))	MOD09030
	X4=H21(1,JT)/X4	MOD09040
	GO TO 6	MOD09050
5	A(4)=H21(1,JT)	MOD09060
	X4=CMPLX(A(4),0.0)	MOD09070
6	DEL=X1*X2-X3*X4	MOD09080
	L=E1(,JT)	MOD09090
	M=E2(1,JT)	MOD09100
	N=E3(1,JT)	MOD09110

M0D09120
M0D09130
M0D09140
M0D09150
M0D09160
M0D09170
M0D09180
M0D09190
M0D09200
M0D09210
M0D09220

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YT(I,L,L)=1./X1
YT(I,L,M)=-1.*Y3/X1
YT(I,M,L)=X4/X1
YT(I,M,M)=DEL/Y1
YT(I,L,L)=1*(YT(I,L,L)+YT(I,L,M))
YT(I,M,M)=-1*(YT(I,M,L)+YT(I,M,M))
YT(I,N,L)=YT(I,L,N)
YT(I,N,M)=YT(I,M,N)
YT(I,N,N)=-1*(YT(I,L,N)+YT(I,M,N))
RETURN
END
SUBROUTINE PRINT
INTEGER HINOD(2),VAR
REAL NEWC(2,10,10),NEWL(2,10,10),NEWC(2,10,10),PN(11,20),
1 P(11,20),F(2,10,11),QN(11,20),Q(11,20),
2 FR(22),DR(22),R(2,10,10),L(2,10,10),C(2,10,10)
COMMON N,N,HINOD,RISE,FALL,VAR,FLON,FIFN,FR,DR,NEWK,NEWL,
1NEWC,R,L,C,IQ,OT/BL1/EL,E2,E3,H11,H11L,H12,H12F,H21,H21F,H22,H22C
2/BL2/I,J,K,J1,Y1,F,W/BL3/ZN,PN,Z,P,QN,Q
WRITE(6,500) IQ
FORMAT(I5,'IQ=',I2)
WRITE(6,501) HINOD(1),HINOD(2)
FORMAT(I5,'HINOD(1)=' ,I2,'HINOD(2)=' ,I2)
WRITE(6,502) N
FORMAT(I5,'N=' ,I2)
WRITE(6,503) M
FORMAT(I5,'M=' ,I2)
IF(IQ-2) 1,2,2
WRITE(6,101)
FORMAT(I33,'NUM' INAL VALUES' ///)
GO TO 3
2 WRITE(6,109) VAR
FORMAT(///,I33,'VARIATION',I3,///)
IF(VAR-1) 303,303,304
303 MV=0
NV=1
304 MV=MV+1
IF(MV-6) 306,305,305
305 MV=MV-5
NV=NV+1
306 RFQ=1.2*(NV-1)*0.4

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M0D00950
M0D00970

M0D04240
M0D04250
M0D04260
M0D04270
M0D04280
M0D04290
M0D04300
M0D04310
M0D04320
M0D04330

500
501
502
503
1
101
2
109
303
304
305
306

```

110 IFQ=2*VAR-I0*(NV-1)-1
3  WRITE(6,110) RFQ,IFQ
   FORMAT(130,'RFQ=',F4.1,I42,'IFQ=',I2,///)
   DO 320 I=1,2
   J=HING0(I)
   DO 320 IX=1,J
   DO 320 IY=1,J
   IF(NEW(I,IX,IY)) 321,322,321
   WRITE(6,401) I,IX,IY,NEW(I,IX,IY)
   FORMAT(15,'NEW(',I1,',',I1,',',I1,')=',IPE13.6)
321 IF(NEWL(I,IX,IY)) 323,324,323
322 WRITE(6,402) I,IX,IY,NEWL(I,IX,IY)
323 FORMAT(15,'NEWL(',I1,',',I1,',',I1,')=',IPE13.6)
324 IF(NEWC(I,IX,IY)) 325,320,325
325 WRITE(6,403) I,IX,IY,NEWC(I,IX,IY)
326 FORMAT(15,'NEWC(',I1,',',I1,',',I1,')=',IPE13.6)
327 CONTINUE
   IF(IQ-2) 4,5,5
4  WRITE(6,102)
102 FORMAT(11,'OF FREQUENCY',I36,'IMPEDANCE',
170,'GT FROM PORT 1 (DB)',//)
   GO TO 6
5  WRITE(6,103)
103 FORMAT(///,I1,'OF FREQUENCY',I19,'IMPEDANCE',I37,'NOM.IMP.-IMP.',
170,'GT FROM PORT 1 (DB)',I81,'NUM.GT-GT(FROM PORT 1)',//)
6  NV = 2*I
   DO 30 J=1,NV
   IF(J-1) 300,171,300
300 IF(J-N) 30,30,172
171 WRITE(6,119) J
119 FORMAT(12,'INPUT PORT',I3)
   GO TO 173
172 JJ=J-N
   IF(JJ-1) 30,187,30
182 WRITE(6,185) J1
185 FORMAT(12,'OUTPUT PORT',I3)
   GO TO 173
183 WRITE(6,181) J1
181 FORMAT(12,'OUTPUT PORT',I3)

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MOD04340
MOD04350
MOD04360

MOD00960
MOD00980
MOD00990

MOD01900
MOD01910
MOD01920
MOD03660
MOD03670
MOD03680
MOD03690
MOD03700
MOD03710
MOD03720
MOD03730
MOD03740
MOD03750
MOD03760
MOD03770
MOD03780
MOD03790

173	CONTINUE	M0003800
	OU 30 K=1,M	M0003810
	IF(J-N) 19,19,0	M0003820
19	FI = F(1,J,K)	M0003830
	GO TO 21	M0003840
20	JNX=J-N	M0003850
	FI = F(2,JNX,K)	M0003860
21	IF(10-1) 25,22,25	M0003870
22	IF(QN(K,J)) 23,24,23	M0003880
23	WRITE(6,104) FI,ZN(K,J),PN(K,J)	M0003890
104	FORMAT(1PE12.4,22X,1PE9.2,1PE9.2,19X,1PE12.2,/))	M0003900
	GO TO 30	M0003910
24	WRITE(6,105) FI,PN(K,J)	M0003920
105	FORMAT(1PE12.4,22X,'INFINITE',28X,1PE12.2,/))	M0003930
	GO TO 30	M0003940
25	DP = PN(K,J)-P(K,J)	M0003950
	IF(QN(K,J)-Q(K,J)) 28,26,29	M0003960
26	IF(QN(K,J)) 29,29,27	M0003970
27	OZ = ZN(K,J)-Z(K,J)	M0003980
	WRITE(6,106) FI,Z(K,J),OZ,P(K,J),DP	M0003990
106	FORMAT(1PE12.4,5X,1PE9.2,1PE9.2,1PE9.2,1PE9.2,5X,1PE9.2,11X,1PE9.2,/))	M0004000
	GO TO 30	M0004010
28	WRITE(6,107) FI,Z(K,J),P(K,J),DP	M0004020
107	FORMAT(1PE12.4,5X,1PE9.2,1PE9.2,1X,'INFINITE',14X,1PE9.2,11X,1PE9.2,/))	M0004030
	GO TO 30	M0004040
29	WRITE(6,108) FI,P(K,J),DP	M0004050
108	FORMAT(1PE12.4,6X,'INFINITE',10X,'INFINITE',14X,1PE9.2,11X,1PE9.2,/))	M0004060
	CONTINUE	M0004070
30	RETURN	M0004080
	END	M0004090
		M0004100

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